The heterogeneous effects of trade on asymmetric countries

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Abstract

In order to investigate the effects of trade on asymmetric countries, we build a simple two-country Melitz Model with cross-country efficiency differences and market size variations. We also introduce a public good financed by Tax revenues. We find that Free Trade makes the more efficient country produce more and alleviate the fiscal burden on its economy,the effects are completely opposite for the other country. Furthermore, we show that when the relative size of the technologically advanced country is large enough, we assist to an agglomeration of resources in this country. The main contribution of this paper is to prove that the more asymmetric the countries are, the more heterogeneous is the impact of Trade on their respective GDP, Aggregate Profits and Tax rate.

1 Introduction

The 20th century ended by putting an end to the era of the standard representative firm assumption in Trade models. The recent emergence of firm level data has paved the way to substantial empirical work which revealed that firms exhibit different levels of productivity even within a narrowly-defined industry and that this tremendous productivity heterogeneity implies selection into exporting. In their seminal papers, Bernard and Jensen (1995,1999) shed light on the fact that within an industry, only a small subset of very efficient firms export.

Moreover, influential empirical findings by Pavcnik(2002) and Tybout(2003) has identified efficiency

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gains from Trade at the industry level. Trade liberalization reallocates market shares from less efficient domestic firms to very efficient foreign exporters. This inter-firm reallocation forces the least productive firms to exit the market, thus increases average productivity at the industry level.

These stylized facts announced the birth of the New New Trade Theory since they could never be explained by previous international trade theories. Classical Trade theories were successful only in explaining inter-industry trade by differences in technology (Ricardian Comparative Advantage) and in relative endowments as in the Heckscher-Ohlin model.

In Krugman(1980), even though the emphasis is on intra-industry trade, firms are identical within an industry and there is no selection into exporting. In response to these empirical challenges, Melitz(2003) provided an extension to the New Trade Theory which features firm heterogeneity and self-selection into producing domestically and exporting due to the presence of sunk fixed costs of entry, production and export.

This New New Trade Theory highlighted three gains from Trade. Trade liberalization reallocates market shares from low productivity domestic suppliers to very productive foreign exporters. This forces the least efficient firms to exit the market and makes high productivity suppliers expand to enter international markets. Thus, average productivity increases at the industry level and the labor force that used to be employed by the least productive firms is henceforth reallocated to the most efficient ones. This efficiency gain implies a variety gain. In fact, market shares reallocation generates a simultaneous increase of the domestic cutoff and decrease of the export cutoff, which raises the probability of export in all countries, thus the Mass of exported varieties. As a result, each country enjoys a variety gain since the loss of domestic varieties is overcompensated by the arrival of much more imported varieties from each one of its trading partners.

Finally, the Welfare gain actually stems from these two gains discussed above. While the increase in average productivity induces a decrease in average price, a larger Mass of available varieties is synonym to a larger Mass of competing firms at the industry level. This clearly leads to a decrease in the Aggregate Price level. As a result, the purchasing power increases since the nominal wage is normalized to 1.

Melitz(2003) describes a perfect world where, not only, all countries gain from trade in terms of efficiency, variety and Welfare, but also, these gains are identical across trading partners. Even though this result mainly stems from the symmetry assumption stating that countries have the same size and identical productivity distribution, trade literature exhibits few trials to investigate the nature of the impact of Trade on asymmetric countries. In fact, far from addressing the asymmetric case, the new wave of heterogeneous firms models, which emerged after the birth of the New New Trade

theory, has only focused on altering some technical assumptions and extending the baseline model in an attempt to reconcile it with new empirical features of firm level data.

In Melitz (2003), each firm produces only a variety of the differentiated good and draws a productivity level that remains unchanged overtime. Moreover, gains from trade arise only at the industry level through inter-firm reallocation. These results are at odds with recent empirical findings showing that trade may lead to substantial firm productivity improvement through intra-firm reallocation of resources. In order to make the benchmark model capture these new features of the data, Mayer, Melitz and Ottaviano (2014) built a heterogeneous firms model with multi-product firms. They find that Trade liberalization improves within-firm productivity and reallocates within-firm factors to the best products since it increases the productivity cutoff to produce a given number of products and makes firm focus on their core competence.

One of the limits of the Melitz model is that Trade liberalization has only a quantity effect reallocating the volume of sales across firms while it has no impact on the prices they set. This rigidity of the pricing rule is explained, not only, by the fact that the productivity level of a firm remains unchanged, but also, by its constant markup implied by CES preferences. To allow markup flexibility, Melitz and Ottaviano(2008) substituted CES by quasi-linear preferences. They find that Trade liberalization is welfare improving since it simultaneously increases average productivity and decreases average markup. Their result illustrates price reactivity to fiercer competition imposed by trade.

Even though this very tractable model has been widely extended, Trade literature remains silent on the nature of the impact of trade on asymmetric countries. The first extension of the Melitz model to the asymmetric countries case was provided by Helpman, Melitz and Yeaple (2003). They assume that countries have an identical productivity distribution and only differ in market size. Using the Home market effect, they show that larger countries enjoy larger mass of available varieties and higher welfare. Their results suggest that bigger countries enjoy higher gains from trade. Building on the work of HMY (2003), Falvey et al. (2004) add an additional feature of country-level asymmetry in terms of productivity distribution. They find that trade liberalization between two countries displaying differences in market size and average productivity improves industry efficiency and welfare in both countries, but this positive impact of trade is more magnified for the more efficient country. These two papers provide two distinct responses to the following question: which country reaps greater gains from trade? The larger, as implicitly suggested by HMY (2003) and the more efficient according to Falvey et al. (2004). Melitz and Ottaviano (2008) provide a unique response which is a combination of both. In their setting characterized by quasi-linear preferences, the larger

country is by construction the more efficient. While all these extensions ensure that all countries gain from trade and emphasize that market size differences or cross-country efficiency gaps only make the gains from exposure to trade accrue disproportionately to the larger and/or more efficient country, a key contribution by Demidova (2008) revealed that trade liberalization generates welfare gains for the technologically advanced country and welfare losses for its trading partner when the technological gap between the two countries is deep enough. Moreover, she highlighted that the impact of trade on welfare in both countries is identical to that of a technological improvement in the more advanced country. More importantly, Demidova (2008) showed that trade deepens the welfare gap between two countries which are already displaying large technological asymmetry.
While the works of Falvey et al.(2004) and Devidova(2008) focused on the heterogeneity of the effect of trade on welfare of asymmetric countries, this paper investigates the nature of the impact of full trade liberalization on GDP, Aggregate Profits and tax rates in countries with different productivity distributions and market sizes.

The paper is organized as follows. Section 2 sets up the model. The closed economy equilibrium is derived in Section 3. Section 4 lays out the properties of the equilibrium in open economy and highlights the impact of free trade. Section 5 studies the determinants of FDI and its effects on the GDP of both countries and section 6 concludes.

2 Set up of the Model

2.1 Public Intervention

We assume that Public Intervention is identical in both countries "A" and "B".In fact, each Government produces L units of the public good and equally distributes it on its population for free. Hence, individual consumption of the public good is rationed to 1.
Both Governments share the same technology, their "unit labor need" is equal to $\frac{1}{L}$.
The Government's "Cost Function" is then written as follow: C(L)=$\frac{1}{L}$L=1
Given that the Nominal wage "w" is equalized across these 2 countries and normalized to 1 (w=1), Public Expenditures are thus equal to 1 for both countries: $G_A = G_B = 1$
Each Government taxes Aggregate Profits made by the private sector to finance its expenditures,its "Budget Constraint" is given by:

-Country A: T_A=t_A Π_A=G_A=1

-Country B: T_B=t_B Π_B=G_B=1

2.2 Demand

A representative consumer consumes Ω varieties of the differentiated good and 1 unit of the Public good. His preferences are represented by a Cobb-Douglas Utility function:
$U = [(\int_\omega q(\omega)^\rho d\omega)^{1/\rho})]^\alpha q(g)^{1-\alpha}$ where $\rho = \frac{\sigma-1}{\sigma}$ and σ=CES between Ω varieties of the differentiated good. Since q(g) is rationed to 1,Utility can then be similar to this of the Baseline Model:
$U = [\int_\omega q(\omega)^\rho d\omega]^{1/\rho}$ and The optimal consumption of a variety ω is: $q(\omega) = Q[\frac{p(\omega)}{P}]^{-\sigma}$

2.3 Supply

2.3.1 Asymmetry

The main feature of Asymmetry between these 2 countries consists in the fact that the initial productivity draw is more favorable for Country A.In other words,the probability to get a a given level of productivity φ is higher for a firm located in Country"A" as compared with another firm established in Country"B".
To illustrate this, we parameterize the "Pareto Distribution" as follow:
Take φ_{max}, $g(\varphi) = (\frac{\varphi}{\varphi_{max}})^k$ such that $g_A(\varphi) = (\frac{\varphi}{\varphi_{max}})^{k_A} > g_B(\varphi) = (\frac{\varphi}{\varphi_{max}})^{k_B}$ with $k_A < k_B$
This implies that $\forall\varphi, G_A(\varphi) < G_B(\varphi)$ where $G(\varphi)$ is the Distribution function.
As result, for the same productivity cutoff φ^*,the probability of a successful entry is higher for firms located in Country"A": $\forall\varphi^*$, $[1 - G_A(\varphi^*)] > [1 - G_B(\varphi^*)]$

2.3.2 Pricing Rule,Revenues and Profits

Optimal price: $\forall$ the country, $p(\varphi) = \frac{1}{\rho\varphi} = \rho^{-1}\varphi^{-1}$
Firm Revenues: $r_A(\varphi) = R_A[\frac{p(\varphi)}{P_A}]^{1-\sigma}$; $r_B(\varphi) = R_B[\frac{p(\varphi)}{P_B}]^{1-\sigma}$
Firm Profit: $\pi_A(\varphi) = \frac{r_A(\varphi)}{\sigma} - f$; $\pi_B(\varphi) = \frac{r_B(\varphi)}{\sigma} - f$

3 Closed Economy Equilibrium

3.1 Conditions of the Equilibrium

1-the Zero Cutoff Profit Condition:

As in Melitz(2003),the Zero Cutoff Profit (ZCP) condition is given by the following equation expressing the average profit $\bar{\pi}$ as a function of the productivity cutoff φ^*:

$$(ZCP) : \bar{\pi} = \pi(\tilde{\varphi}) = fk(\varphi^*) \tag{1}$$

The principle of this condition is available for both countries, thus the (ZCP) condition is identical for these two countries.

2- The Free Entry Condition:

$$(FE) : \bar{\pi} = \frac{\delta fe}{[1 - G(\varphi^*)]} \tag{2}$$

Knowing that for a given cutoff level φ^*,the probability of success is higher for plants established in Country"A",the equation above (FE) implies that the average profit $\bar{\pi}$ is lower in this country,which is quite intuitive because competition is fiercer between relatively more efficient firms as compared with Coutry"B".This can be expressed as follow: Since $\forall\varphi^*, [1 - G_A(\varphi^*)] > [1 - G_B(\varphi^*)], \bar{\pi_A} < \bar{\pi_B}$ Graphically,this means the (FE) curve of Country"A" is below this of Country"B", yielding a higher productivity cutoff in Country"A": $\varphi_A^* > \varphi_B^*$ as illustrated in the figure below:

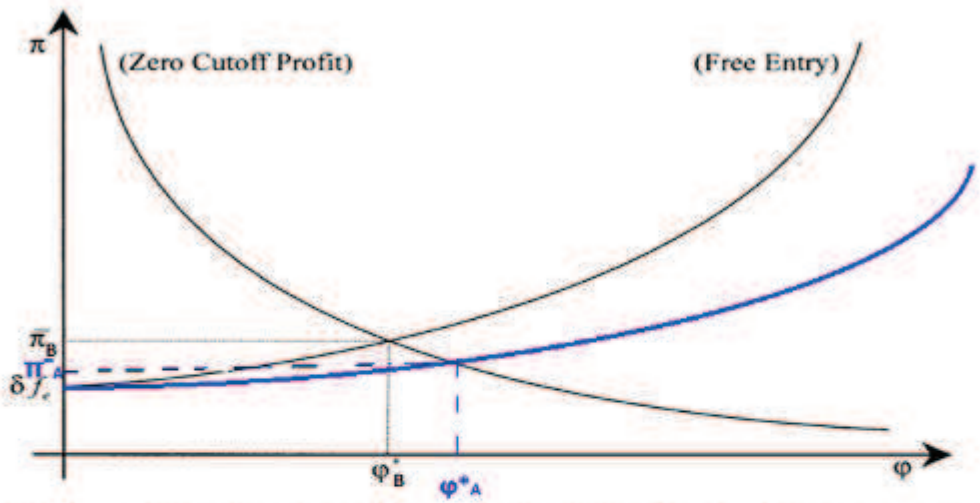


FIGURE 1.—Determination of the equilibrium cutoff φ^* and average profit $\bar{\pi}$.

3.2 Equilibrium Mass of firms,GDP and Aggregate Profits

We assume that both countries have the same size: $\lambda = \frac{L_A}{L_B} = \frac{R_A}{R_B} = 1$.

The Mass of firms in each country are given by: $M_A = \frac{R_A}{\bar{r_A}} = \frac{L_A}{\sigma(\bar{\pi_A}+f)}$ and $M_B = \frac{R_B}{\bar{r_B}} = \frac{L_B}{\sigma(\bar{\pi_B}+f)}$

We have to note that even if the market size is identical for both countries,the Mass of firms is higher in Country"A" since the average profit is lower in this more competitive country: for $L_A = L_B$, $M_A > M_B$ since $\bar{\pi}_A < \bar{\pi}_B$.

Countries'Outputs can be computed as follow:

$GDP_A = M_A\tilde{q_A}$ where $\tilde{q_A} = R_A(\frac{p(\bar{\varphi_A})}{P_A})^{-\sigma}$ is the average output per firm in Country"A"

$GDP_B = M_B\tilde{q_B}$ where $\tilde{q_B} = R_B(\frac{p(\bar{\varphi_B})}{P_B})^{-\sigma}$ is, similarly, the output of an average productivity firm in Country"B".

Aggregate Profits are written as follow:$\Pi_A = M_A\bar{\pi_A}$ and $\Pi_B = M_B\bar{\pi_B}$

For more simplicity,we equalize GDP and Aggregate Profits using a condition stating that if the Mass of firms is x percent higher is Country"A", average profit and average output are then x percent higher in Country"B": GDP_A=GDP_B and $\Pi_A = \Pi_B$ iff $\frac{M_A}{M_B}$=$\frac{\bar{\pi_B}}{\bar{\pi_A}}$= $\frac{\tilde{q}_B}{\tilde{q}_A}$

3.3 Equilibrium Tax rates

Each Government chooses a tax rate "t" that balances its budget:

$Gov'_A s\ budget\ constraint : t_A \Pi_A = 1 \Rightarrow t_A^* = \frac{1}{\Pi_A}$

$Gov'_B s\ budget\ constraint : t_B \Pi_B = 1 \Rightarrow t_B^* = \frac{1}{\Pi_B}$

Aggregate Profits are equalized across these 2 countries, thus the equilibrium tax rate is identical for both countries: t_A^*=t_B^*

3.4 Analysis of the Equilibrium

Average productivity in each country is written as follow:

$\tilde{\varphi}_A = [\int_{\varphi_A^*}^{\varphi_{max}} \varphi^{\sigma-1} \mu_A(\varphi) d\varphi]^{\frac{1}{\sigma-1}}$ and $\tilde{\varphi}_B = [\int_{\varphi_B^*}^{\varphi_{max}} \varphi^{\sigma-1} \mu_B(\varphi) d\varphi]^{\frac{1}{\sigma-1}}$

Since the productivity cutoff is higher in Country"A", average productivity is higher in this country: $\varphi_A^* > \varphi_B^* \Rightarrow \tilde{\varphi}_A > \tilde{\varphi}_B$. Moreover, given that Country"A" hosts a lager Mass of firms, the Aggregate productivity is then higher in this country: $\tilde{\varphi}_A > \tilde{\varphi}_B$ and $M_A > M_B \Rightarrow \phi_A = M_A \tilde{\varphi_A} > \phi_B = M_B \tilde{\varphi_B}$. The Aggregate Price level $P = M^{\frac{1}{(1-\sigma)}} p(\tilde{\varphi})$ where $p(\tilde{\varphi}) = \rho^{-1} \tilde{\varphi}^{-1}$ is a decreasing function of the Mass of firms "M" and an increasing function of the average price"$p(\tilde{\varphi})$. Not only, the Mass of firms, but also, the average productivity is higher in Country"A", thus The Aggregate Price level is lower in this Country: $P_A < P_B$.

Recall that the Nominal Wage "w" is equalized across countries and normalized to 1. This implies that consumers in Country"A" enjoy a higher purchasing power and a larger set of available varieties, which is synonym of higher Welfare.

This result mirrors the "Home efficiency effect" reported in Falvey et al(2004) and Demidova(2008). Even if these two countries have the same size, average productivity is higher in the more efficient country. This implies, not only, a larger Mass of firms, but also, a lower average price. As a result, consumers in this country enjoy higher welfare. This is also in line with the findings of Helpman,Melitz and Yeaple(2003) who showed that larger countries enjoy higher Welfare. However, we have to note that their results are solely driven by the "Home market effect". In Melitz and Ottaviano(2008), a larger country is more efficient by construction. Thus, its higher welfare is explained by its larger variety range, higher average productivity and lower average markup.

4 Open Economy Equilibrium

In open economy, the two countries create a Currency Union. As a result, they freely trade and the Nominal wage remain equalized across these two members due to free labor mobility.

4.1 Export Revenues versus Domestic Revenues

In Melitz(2003): $r_d(\varphi) = R(\frac{p(\varphi)}{P})^{1-\sigma}$ and $r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi)$. The export revenues are function of the domestic revenues simply because by symmetry, the Aggregate Revenues"R" and the Aggregate Price"P" are identical across countries. However, in this extension of the baseline model, even if we have assumed that both countries have the same size(L_A=L_B), thus the same level of Aggregate expenditure(R_A=R_B=R), the Aggregate Price level remains lower in Country"A":$P_A < P_B$.

It is then intuitive to express the domestic revenues as a function of the Demand characteristics of the domestic country and the export revenues as a function of the Demand determinants of the Destination country, as follow:
$r_d^A(\varphi) = R(\frac{p(\varphi)}{P_A})^{1-\sigma}$ and $r_x^A(\varphi) = R\tau^{1-\sigma}(\frac{p(\varphi)}{P_B})^{1-\sigma}$

$r_d^B(\varphi) = R(\frac{p(\varphi)}{P_B})^{1-\sigma}$ and $r_x^B(\varphi) = R\tau^{1-\sigma}(\frac{p(\varphi)}{P_A})^{1-\sigma}$
Using the export revenues equations above, we can easily notice that for the same level of productivity and the same export price, exporters from Country"A" have a lower relative export price on their destination market(Country"B") which is characterized by a higher Aggregate Price level. In other words, Country A's exporters enjoy a higher international price competitiveness as compared to those of Country B, thus they generate higher export revenues and profits as shown below:
$\frac{r_x^A(\varphi)}{r_x^B(\varphi)} = \frac{\pi_x^A(\varphi)}{\pi_x^B(\varphi)} = (\frac{P_B}{P_A})^{\sigma-1} > 1$

4.2 Equilibrium Export Cutoffs

Assuming that the export cutoff is identical in both countries, we can intuitively infer,as proved above, that the export cutoff revenues are higher for exporters from Country"A". This means that while the export cutoff profit is equal to zero(as the definition suggests) for country B's exporters, it is strictly positive for Country A's exporters. As a result the export cutoff has to be lower in Country A such that it equalizes the relative cutoff export price across countries, which ensures that the export cutoff profit is identical for exporters from both countries and equal to zero:

$\varphi_x^{*A} < \varphi_x^{*B}$ such that $(\frac{p(\varphi_x^{*A})}{P_B})=(\frac{p(\varphi_x^{*B})}{P_A}) \Rightarrow r_x^A(\varphi_x^{*A})=r_x^B(\varphi_x^{*B}) = \sigma f_x \Rightarrow \pi_x^A(\varphi_x^*) = \pi_x^B(\varphi_x^*) = 0$
For proof by contradiction, see Appendix 1.1

4.3 Average Export Revenues and Profits

The fact that export cut off is lower in Country A means that average export productivity is lower in this country. This implies that the average export price is higher in this country, but once deflated by a higher Aggregate Price level in the destination country(B), relative average export prices are equalized across these two countries. Hence, average export revenues and profits are identical for exporters from both countries:
$\varphi_x^{*A} < \varphi_x^{*B} \Rightarrow \tilde{\varphi}_x^A < \tilde{\varphi}_x^B \Rightarrow p(\tilde{\varphi}_x^A) > p(\tilde{\varphi}_x^B) \Rightarrow (\frac{p(\tilde{\varphi}_x^A)}{P_B}) = (\frac{p(\tilde{\varphi}_x^B)}{P_A})$
$\Rightarrow r_x^A(\tilde{\varphi}_x^A) = r_x^B(\tilde{\varphi}_x^B) = \bar{r}_x \Rightarrow \pi_x^A(\tilde{\varphi}_x^A) = \pi_x^B(\tilde{\varphi}_x^B) = \bar{\pi}_x$

4.4 Probability of Export and Mass of Exporters

As in Melitz(2003), we define the probability of export in each country as follow:
$P_x^A = [1 - G_A(\varphi_x^{*A})]/[1 - G_A(\varphi_A^*)]$
$P_x^B = [1 - G_B(\varphi_x^{*B})]/[1 - G_B(\varphi_B^*)]$
As compared with country B, Country A has, not only, a higher domestic cutoff, but also, a lower export cutoff. Hence, the probability of export is higher in this country, which hosts also a larger Mass of firms. Thus, Country A has a higher Mass of exporters:
$P_x^A > P_x^B$ and $M_A > M_B \Rightarrow M_x^A = P_x^A M_A > M_x^B = P_x^B M_B$

4.5 Mass of Exiting Firms

In the baseline model, Marc Melitz, using simple static comparatives, shows that a firm with a higher productivity level generates higher revenues as compared with a less efficient firm. This implies that 2 firms have the same level of revenues only and only if their initial productivity level is identical:
$\frac{r(\varphi')}{r(\varphi)} = (\frac{\varphi'}{\varphi})^{\sigma-1} \Rightarrow r(\varphi') = r(\varphi)$ iff $\varphi'=\varphi$
In subsection(4.3), we equalized average export revenues across countries, which implicitly induces that average export productivity could be identical for both countries:
$r_x^A(\tilde{\varphi}_x^A) = r_x^B(\tilde{\varphi}_x^B) = \bar{r}_x \Rightarrow \tilde{\varphi}_x^A = \tilde{\varphi}_x^B = \tilde{\varphi}_x$
In order to quantify the pro-competitive effect of Trade,we use the same static comparatives tech-

nique as in Melitz(2003). In fact, we compare the revenues of an average productivity exporter to those of an average productivity domestic firm on the destination market. Hence, we easily determine, for each country, the number of domestic plants that are forced to exit the market due to the entry of 1 exporter as follow:

$\frac{r_x^A(\tilde{\varphi}_x^A)}{r_d^B(\tilde{\varphi_B})} = (\frac{\tilde{\varphi}_x}{\tau\tilde{\varphi}_B})^{\sigma-1} = \theta_A$; $\frac{r_x^B(\tilde{\varphi}_x^B)}{r_d^A(\tilde{\varphi_A})} = (\frac{\tilde{\varphi}_x}{\tau\tilde{\varphi}_A})^{\sigma-1} = \theta_B$

These static comparatives reveal that the pro-competitive effect of Trade is more magnified for Country"B". In fact, the number of exiting domestic firms per average exporter(θ) is higher in this country: $\tilde{\varphi}_B < \tilde{\varphi}_A \Rightarrow \theta_A > \theta_B$. Multiplying this number by the Mass of exporters, we get the Mass of Exiting firms in each country: $M_{EX}^B = \theta_A M_x^A > M_{EX}^A = \theta_B M_x^B$.

4.6 Equilibrium Mass of Firms and Average Profit in Open Economy

In each country, the Mass of firms in open economy is actually the Mass of firms that survive after full liberalization of Trade. In other words,it is equal to the equilibrium Mass of firms in closed economy net of the Mass of exiting firms.

In open economy,the Mass of firms remains larger in Country"A", not only, because it was already larger in closed economy,but also,due to the fact that the pro-competitive of Trade is less magnified for this country, implying a smaller Mass of exiting firms:

$M_A^O = M_A - M_{EX}^A > M_B^O = M_B - M_{EX}^B$ since $M_A > M_B$ and $M_{EX}^A < M_{EX}^B$

Average Profit in open economy is computed as follow:

$\bar{\pi}_A^O = \bar{\pi}_A + P_x^A\ \bar{\pi}_x - P_x^B\ \bar{\pi}_x = \bar{\pi}_A + \bar{\pi}_x\ (P_x^A - P_x^B)$

$\bar{\pi}_B^O = \bar{\pi}_B + P_x^B\ \bar{\pi}_x - P_x^A\ \bar{\pi}_x = \bar{\pi}_B + \bar{\pi}_x\ (P_x^B - P_x^A)$

Given that the probability of export is higher in Country"A" and average export profit is equalized across countries, these 2 equations above show that while Free Trade increases average profitability in the more efficient country(A),it makes firms in the other country(B),on average,less profitable.

4.7 GDP and Aggregate Profits in Open Economy

To assess the impact of Trade on the Output and the Aggregate Profit in each trading country, we simply compare the value of these variables in Open economy to those obtained in Closed economy. The equations below confirm that Free Trade makes the more efficient country(A) produce more and enjoy higher Aggregate Profits and that these effects are completely opposite for the other Country(B):

$*GDP_A^O = GDP_A + M_x^A\ \tilde{q}_x^A - M_{EX}^A\ \tilde{q}_A$

$= GDP_A + P_X^A\ M_A\ [R_B(\frac{p(\tilde{\varphi}_x^A)}{P_B})^{-\sigma}] - P_x^B\ M_B\ \theta_B\ [R_A(\frac{p(\tilde{\varphi}_A)}{P_A})^{-\sigma}]$

$\Rightarrow \Delta\ GDP_A > 0$

Under the following "Stability Condition": $\frac{d\tilde{q}_A}{dL_A} = 0$

(See Appendix for more details), we can write that:$\Delta GDP_A = f^+(M_A)$

$*GDP_B^O = GDP_B + M_X^B\ \tilde{q}_x^B - M_{EX}^B\ \tilde{q}_B$

$= GDP_B + P_X^B\ M_B\ [R_A(\frac{p(\tilde{\varphi}_x^B)}{P_A})^{-\sigma}] - P_x^A\ M_A\ \theta_A\ [R_B(\frac{p(\tilde{\varphi}_B)}{P_B})^{-\sigma}]$

$\Rightarrow \Delta\ GDP_B < 0$

Note that $\Delta\ GDP_B = f^-(M_A)$ if this "Stability Condition" is verified:$\frac{d\tilde{q}_x^B}{dL_A} = 0$

$*\Pi_A^O = M_A^O\ \bar{\pi}_A^O = (M_A - M_{EX}^A)\ [\bar{\pi}_A + \bar{\pi}_x(P_x^A - P_x^B)]$

$= (M_A\bar{\pi}_A) + M_A\bar{\pi}_x(P_x^A - P_x^B) - M_{EX}^A\bar{\pi}_A - M_{EX}^A\bar{\pi}_x(P_x^A - P_x^B)$

$\Rightarrow \Delta\Pi_A = (M_A - M_{EX}^A)\ \bar{\pi}_x(P_x^A - P_x^B) - M_{EX}^A\bar{\pi}_A$

$= M_A^O\bar{\pi}_x(P_x^A - P_x^B) - M_{EX}^A\bar{\pi}_A > 0$

$\Delta\Pi_A = f^+(M_A)$ under the following "Stability Condition": $\frac{d\tilde{r}_x^B}{dL_A} = 0$

$*\Pi_B^O = M_B^O\ \bar{\pi}_B^O = (M_B - M_{EX}^B)[\bar{\pi}_B + \bar{\pi}_x(P_x^B - P_x^A)]$

$= (M_B\bar{\pi}_B) + M_B\bar{\pi}_x(P_x^B - P_x^A) - M_{EX}^B\bar{\pi}_B^O$

$\Rightarrow \Delta\Pi_B = -M_B\bar{\pi}_x(P_x^A - P_x^B) - M_{EX}^B\ \bar{\pi}_B^O < 0$

Note that $\Delta\Pi_B = f^-(M_A)$ since $\Delta\Pi_B = f^-(M_{EX}^B)$ and $M_{EX}^B = \theta_A P_x^A M_A$

These equations reveal that the variations of the Output and Aggregate Profits in Country"A" are an increasing function of its size(L_A). However, Country B's Output and Aggregate profits would react more negatively to full liberalization of Trade if the size of their trading partner (L_A) was larger. For more details, see Appendix 1.2

4.8 Tax rate Adjustment

Given that Public Expenditures are identical across countries and equal to 1 and that each Government has to balance its budget, an increase/decrease in Aggregate Profits leads to a decrease/increase, with the same magnitude, of the tax rate.

As a result, while Country A, which enjoys higher Aggregate Profits in Open economy, alleviates the fiscal burden on its firms, Country B taxes more its economy after full openness to Trade to

compensate the decline in Aggregate Profits, as shown below:

Recall that the Gov's budget constraint in each country is written:

Country"A": $t_A\Pi_A$= 1 and Country"B": $t_B\Pi_B$= 1

Taking Logs: $Ln(t_A)$= - $ln(\Pi_A) < 0$ and $Ln(t_B)$= - $ln(\Pi_B) > 0$

Hence, the equilibrium Tax rates in Open Economy are given by:

$t_A^{*'}= t_A^* + Ln(t_A) \Rightarrow t_A^{*'} < t_A^*$

$t_B^{*'}= t_B^* + Ln(t_B) \Rightarrow t_B^{*'} > t_B^*$

4.9 Analysis of the impact of Free Trade

Starting from a "Closed Economy Equilibrium" where both countries have the same size(λ=$\frac{L_A}{L_B}$=1 and $GDP_A = GDP_B$), the same level of Aggregate Profits($\frac{\Pi_A}{\Pi_B} = 1$) and the same tax rate($\frac{t_A^*}{t_B^*} = 1$), we have shown that Free Trade makes the more efficient country(A) produce more($GDP_A^O > GDP_B^O$) and enjoy a relatively higher Aggregate Profit($\frac{\Pi_A^O}{\Pi_B^O} > 1$) and a relatively lower Tax rate($\frac{t_A^{*'}}{t_B^{*'}} < 1$). Nevertheless, the variety gain is higher for Country B. In fact, its consumers enjoy a larger Mass of additional varieties coming from Country A since this country has a higher Mass of exporters.

4.10 The Home Market effect still matters!

We have already proved that when both countries have the same size(λ=1), only Country A gains from Trade in terms of production, Profitability and lower taxation. We have also shed light on the fact that its gains,which are synonym of Losses for its trading partner(Country B), are an increasing function of its size(L_A).This implies that the variations of the relative GDP of Country A, the relative Aggregate Profits of this country and the relative tax rate of Country B are all an increasing function of the relative size of country A(λ). In other words, the larger the relative size of Country A, the higher its gains and the bigger the losses for country B.

Hence, the Market size amplifies the initial impact of Trade on both countries!

5 FDI

5.1 FDI,the Trade-off

We have previously shown that the higher the relative Market size of Country"A"(λ), the higher the relative fiscal pressure in Country"B"($\frac{t_B}{t_A}$).

Firms located in the less efficient Country(B) are facing the following Trade-off:

-Either continue to produce locally,thus avoid paying the fixed FDI cost (F_i) but accept to be relatively more taxed on their profits.

-Or pay (F_i), relocate their production to Country A and enjoy lower fiscal pressure.

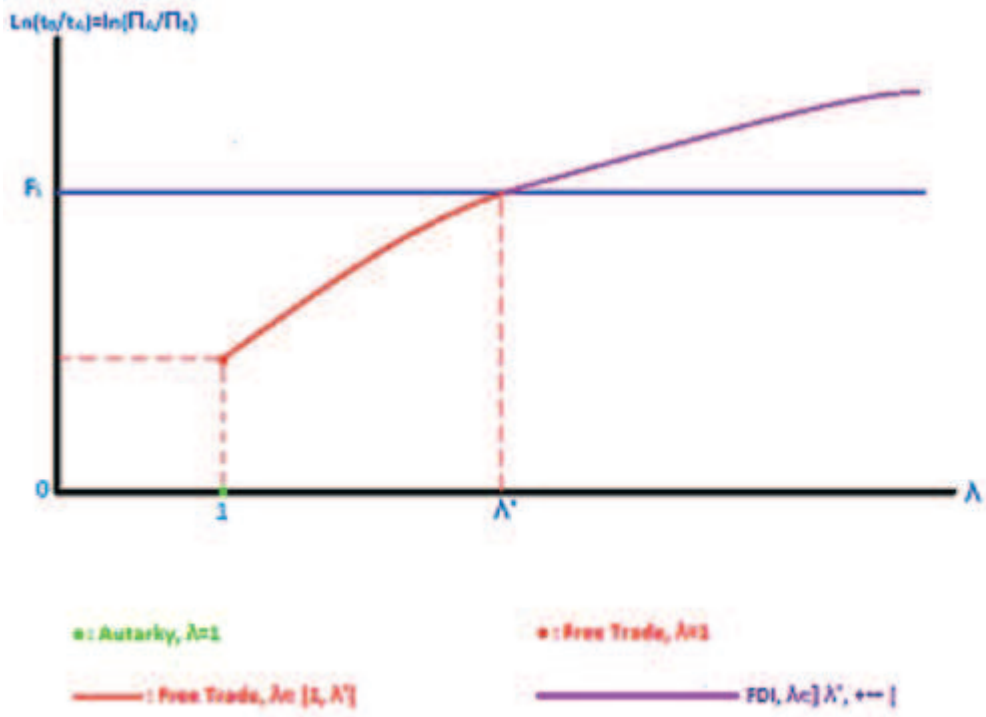
5.2 FDI function

$FDI_B^A = \ln(\frac{t_B}{t_A})$ - F_i

FDI from Country B to Country A is an increasing function of the relative size of this latter(λ). In fact, the higher λ,the higher the relative Aggregate Profit in Country A and the higher the relative tax rate in Country B. Thus, the stronger is the incentive for firms located in Country B to relocate their production to the other Country to benefit from a more favorable tax regime.

$FDI_B^A = \ln(\frac{t_B}{t_A})$ - $F_i > 0 \; \forall \; \lambda > \lambda^*$

λ^* represents the relative Market size cutoff beyond which the gain from lower taxation in Country A exceeds the fixed FDI cost F_i. Therefore, when λ is high enough ($\lambda > \lambda^*$), the fiscal burden becomes so heavy in Country B that all the firms escape it through relocating their entire production to Country A where the huge gains from Tax alleviation exceed by far the fixed FDI cost. This is clearly illustrated in the figure below:



5.3 GDP reaction to Relative Market Size

When the relative market size of Country A is not too high ($\lambda < \lambda^*$), the tax rate is relatively lower in Country A but the gain from fiscal alleviation is not that large to compensate the fixed FDI cost. Hence, there is no FDI and countries freely trade only. Their respective Outputs in Open economy can be written as follow:

$\forall\lambda \in [1, \lambda^*[$, we have:$\forall\lambda \in [1, \lambda^*[$, we have:

$GDP_A^O = GDP_A + \Delta GDP_A(^+\lambda)$

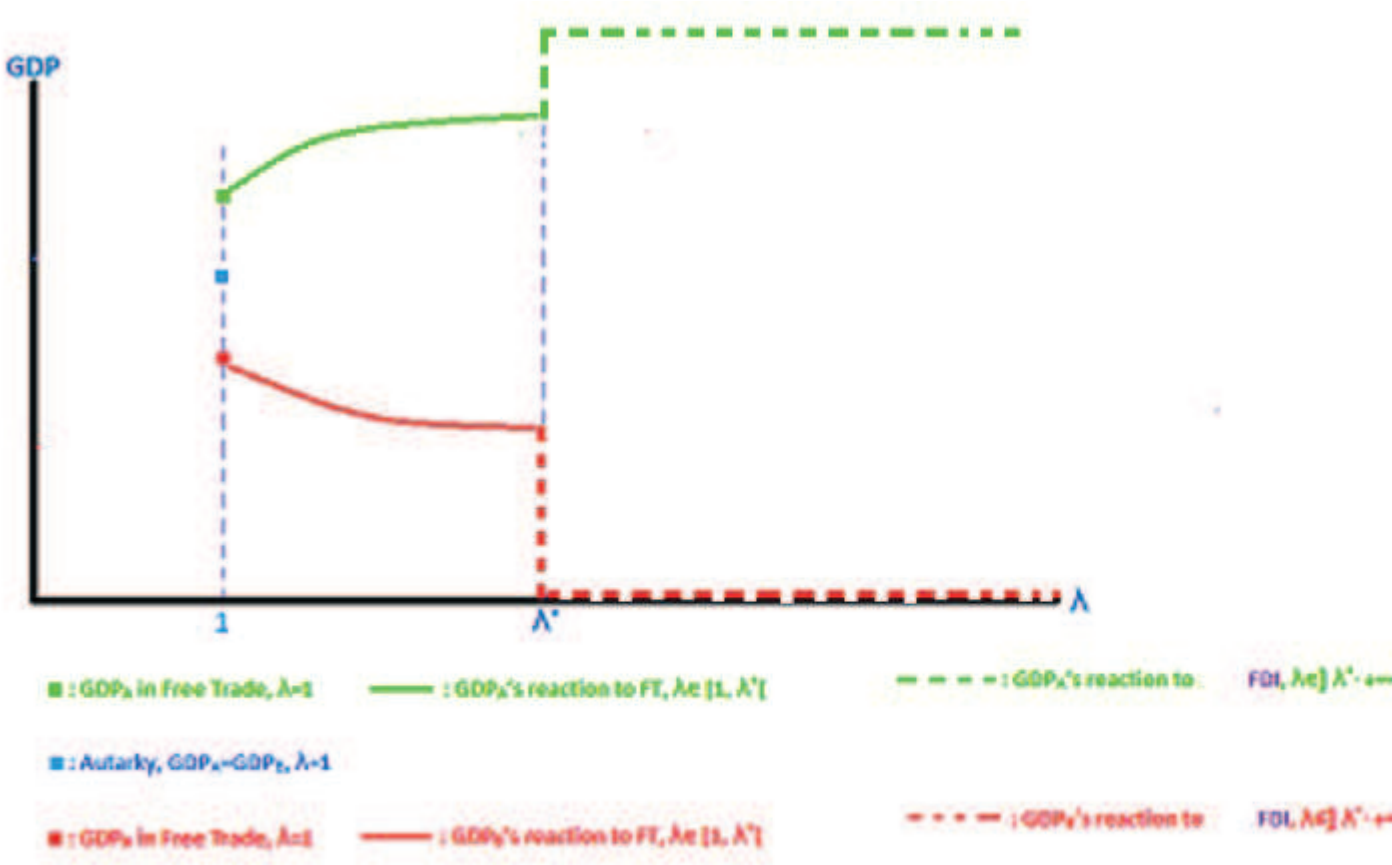
$GDP_B^O = GDP_B + \Delta GDP_B(^-\lambda)$

However,when λ is high enough ($\lambda > \lambda^*$), taxation becomes so favorable in Country A that all the firms of Country B decide to relocate their production to Country A. In other words, the more efficient Country(A) fully absorbs its trading partner through FDI as shown below:

$\forall\lambda \in [\lambda^*, +\infty[$, and since $FDI_B^A = GDP_B$ we get :

$^*GDP_A^O = GDP_A + FDI_B^A = GDP_A + GDP_B$

$^*GDP_B^O = GDP_B - FDI_B^A = 0$



6 Conclusion

This simple model reveals that Free Trade between 2 countries having the same size(L) but different productivity distribution makes the more efficient efficient country gain(produce more, enjoy higher Aggregate Profit and alleviate fiscal pressure on its economy) and the other country lose(produce less and tax more its economy to compensate the decline of Aggregate Profits).

Moreover, using the "Home Market effect",we noticed that The higher the relative size of the more efficient country, the higher its gains from Trade and the larger the losses of the other country.

Furthermore, we shed light on the fact that the more efficient country can fully absorb its trading partner through FDI when its relative size is beyond the cutoff λ^*.

To sum up, the main contribution of this paper is to prove that the more Asymmetric the countries are, the more heterogeneous the impact of Trade is !

7 Appendix

7.1 Appendix 1.1: Proof by Contradiction:

Let's assume that φ_x^{*A}=φ_x^{*B}=φ_x^* and compare $r_x^A(\varphi_x^*)$ to $r_x^B(\varphi_x^*)$:

$\frac{r_x^A(\varphi_x^*)}{r_x^B(\varphi_x^*)} = (\frac{P_B}{P_A})^{\sigma-1} > 1 \Rightarrow r_x^A(\varphi_x^*) > r_x^B(\varphi_x^*)$=$\sigma f_x$

$\Rightarrow \pi_x^A(\varphi_x^*) > \pi_x^B(\varphi_x^*) = 0$

Hence, φ_x^{*A} has to be $< \varphi_x^{*B}$ such that $(\frac{p(\varphi_x^{*A})}{P_B})$=$(\frac{p(\varphi_x^{*B})}{P_A})$

This ensures that $r_x^A(\varphi_x^{*A})$=$r_x^B(\varphi_x^{*B}) = \sigma f_x$. Thus, $\pi_x^A(\varphi_x^*) = \pi_x^B(\varphi_x^*) = 0$

7.2 Appendix 1.2: Stability Conditions

$\tilde{q}^A$, $\tilde{q}_x^B$ and $\tilde{r}_x^B$ are increasing functions of the Aggregate Revenues (R_A) and the Aggregate Price level (P_A) in country"A".
An increase in the Market size of Country"A" (L_A) has 2 opposite effects on these 2 determinants:

-it increases the Aggregate Revenues (R_A), which shifts upward $\tilde{q}^A$, $\tilde{q}_x^B$ and $\tilde{r}_x^B$

-it decreases the Aggregate Price level (P_A) because it implies a larger Mass of firms [$M = f^+(L)$ and $P = f^-(M)$].

Hence, the relative price $(\frac{p(\varphi)}{P_A})^{-\sigma}$ increases,which weakens the price-competitiveness and shifts downward $\tilde{q}^A$, $\tilde{q}_x^B$ and $\tilde{r}_x^B$

We assume for simplicity that these 2 effects cancel out by writing : $\frac{d\tilde{q}_A}{dL_A} = \frac{d\tilde{q}_x^B}{dL_A}$=$\frac{d\tilde{r}_x^B}{dL_A} = 0$

This assumption allows us to isolate the effect of relative market size variation on the magnitudes of gains/losses in Country A/Country B.

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